



U8432

Reg. No.: .....

Name:.....

**University of Kerala**

First Semester Degree Examination, November 2024

Four Year Under Graduate Programme

Discipline Specific Core Course

**Mathematics****UK1DSCMAT107 Relation, Functions and Number theory**

Academic Level: 100-199

**Time:2 Hours****Max.Marks:56****Part A.**

Answer All Questions Objective Type. 1 Mark Each.

(Cognitive Level: Remember/Understand)

**6 Marks.** Time: 5 Minutes

<b>Qn. No.</b>	<b>Question</b>	<b>Cognitive Level</b>	<b>Course Outcome (CO)</b>
1.	Give an example of a lattice.	Remember	CO 2
2.	When a relation can be a universal relation.	Remember	CO 1
3.	Give an example of a reflexive relation.	Understand	CO 1
4.	Find the coefficient of highest degree term in the expansion of $(2-3x)^5$ .	Understand	CO 3
5.	Find $\sigma(20)$ .	Understand	CO 4
6.	Find all equivalence classes in the relation defined on the integers as $a R b$ if and only if $a=b$	Understand	CO 2

**Part B.**

Answer All Questions Two-Three sentences. 2 Marks Each.

(Cognitive Level: Remember/Understand/Apply)

**10 Marks.** Time: 20 Minutes

<b>Qn. No.</b>	<b>Question</b>	<b>Cognitive Level</b>	<b>Course Outcome (CO)</b>
7.	Check whether the function $f: R \rightarrow R$ defined by $f(x)=2x-3$ is a bijection.	Understand	CO 1
8.	If $a \vee bc$ , with $\gcd(a, b) = 1$ , then prove that $a \vee c$ .	Understand	CO 4

9.	Write the dual of $A = (B \cap A) \cup (A \cap B)$ where A and B are sets.	Apply	CO 1
10.	Let $f: R \rightarrow R \wedge g: R \rightarrow R$ be defined as $f(x) = x^2 \wedge g(x) = 2x + 3$ . Determine fog.	Apply	CO 2
11.	Find the least upper bound of poset $(\{1,2,4,8,16\}, l)$ where $l$ means "divisor of".	Apply	CO 3

### Part C.

Answer all 4 questions, choosing among options within each question.

Short Answer. 4 Marks Each.

(Cognitive Level: Remember/Understand/Apply/Analyse)

**16 Marks.** Time: 35 Minutes

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
12.	<p>A) If R is the relation on the set of positive integer such that <math>(a, b) \in R</math> if and only if <math>a + b = 3n</math> for some integer n, prove that R is an equivalence relations</p> <p>OR</p> <p>B) For the sets A, B, C prove or disprove the statement <math>A \cap C = B \cap C</math> and <math>A - C = B - C \rightarrow A = B</math> with proper justification.</p>	Apply	CO 1
13.	<p>A) Check whether the relation R represented by <math>M_R = \begin{bmatrix} 1 &amp; 0 &amp; 1 \\ 0 &amp; 1 &amp; 0 \\ 1 &amp; 0 &amp; 1 \end{bmatrix}</math> is an equivalence relation.</p> <p>OR</p> <p>B) Draw the digraph representing the partial ordering <math>\{(a, b): a b\}</math> on the set <math>\{1, 2, 3, 4, 5, 6, 7, 8\}</math>. Reduce it into the Hasse diagram representing the given partial ordering.</p>	Apply	CO 2
14.	<p>A) Prove that <math>1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}</math></p> <p>OR</p> <p>B) Let a and b be integers, not both zero. Prove that a and b are relatively prime if and only if there exist integers x and y such that <math>1 = ax + by</math>.</p>	Apply	CO 3
15.	<p>A) State and prove Wilson's Theorem.</p> <p>OR</p> <p>B) Prove that <math>1^p + 2^p + 3^p + \dots + (p-1)^p</math> is a multiple of p.</p>	Analyse	CO 4

**Part D.**

Answer all 4 questions, **choosing among options within each question.**

Long Answer. 6 Marks Each.

(Cognitive Level: Understand/Apply/Analyse/Evaluate/Create)

**24 Marks.**

Time: 60 Minutes

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
16.	<p>A) Define Partial ordered set with an example. Draw the Hasse diagram of the relation 'x divides y' on the set <math>A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}</math></p> <p>OR</p> <p>B) Let <math>(L, \leq)</math> be a lattice. Prove that for any <math>a, b, c \in L</math></p> <p>(i) <math>a \wedge b = b \wedge a</math></p> <p>(ii) <math>a \vee (b \vee c) = (a \vee b) \vee c</math></p>	Analyse	CO 1
17.	<p>A) State and prove fundamental theorem of arithmetic.</p> <p>OR</p> <p>B) Prove that for all <math>n \geq 1</math>,</p> $2 \cdot 6 \cdot 10 \cdot 14 \dots (4n-2) = \frac{(2n)!}{n!}$	Evaluate	CO 3
18.	<p>A) Using Warshal's algorithm to find the transitive closure of a relation <math>R = \{(1,2), (2,3), (3,4), (2,1)\}</math> for the set <math>A = \{1, 2, 3, 4\}</math>.</p> <p>OR</p> <p>B) If <math>R</math> is the relation on <math>A = \{3,4,5,6\}</math> Such that <math>(a,b) \in R</math> iff <math>a-b = \text{even}</math>. Find the rational matrix <math>M_R</math>. find also the rational matrices of <math>R^{-1}</math>, <math>\dot{R}</math> and <math>R^2</math>.</p>	Apply	CO 2
19.	<p>A) Find the remainder when <math>2^{340}</math> is divided by 341.</p> <p>OR</p> <p>B) Show that <math>18! + 1 \equiv 0 \pmod{437}</math>.</p>	Apply	CO 4